NAG Toolbox for MATLAB

f08se

1 Purpose

f08se reduces a real symmetric-definite generalized eigenproblem $Az = \lambda Bz$, $ABz = \lambda z$ or $BAz = \lambda z$ to the standard form $Cy = \lambda y$, where A is a real symmetric matrix and B has been factorized by f07fd.

2 Syntax

```
[a, info] = f08se(itype, uplo, a, b, 'n', n)
```

3 Description

To reduce the real symmetric-definite generalized eigenproblem $Az = \lambda Bz$, $ABz = \lambda z$ or $BAz = \lambda z$ to the standard form $Cy = \lambda y$, f08se must be preceded by a call to f07fd which computes the Cholesky factorization of B; B must be positive-definite.

The different problem types are specified by the parameter **itype**, as indicated in the table below. The table shows how C is computed by the function, and also how the eigenvectors z of the original problem can be recovered from the eigenvectors of the standard form.

itype	Problem	**uplo**	B	C	z
1	$Az = \lambda Bz$	'U' 'L'	$U^{\mathrm{T}}U$ LL^{T}	$U^{-\mathrm{T}}AU^{-1}$ $L^{-1}AL^{-\mathrm{T}}$	$U^{-1}y$ $L^{-\mathrm{T}}y$
2	$ABz = \lambda z$	'U' 'L'	$U^{\mathrm{T}}U$ LL^{T}	UAU^{T} $L^{\mathrm{T}}AL$	$U^{-1}y$ $L^{-\mathrm{T}}y$
3	$BAz = \lambda z$	'U' 'L'	$U^{\mathrm{T}}U$ LL^{T}	UAU^{T} $L^{\mathrm{T}}AL$	$U^{\mathrm{T}}y$ Ly

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **itype – int32 scalar**

Indicates how the standard form is computed.

itype $= 1$

if **uplo** $=$ 'U', $C = U^{-\mathrm{T}}AU^{-1}$;
if **uplo** $=$ 'L', $C = L^{-1}AL^{-\mathrm{T}}$.

itype $= 2$ or 3

if **uplo** $=$ 'U', $C = UAU^{\mathrm{T}}$;
if **uplo** $=$ 'L', $C = L^{\mathrm{T}}AL$.

Constraint: **itype** $= 1$, 2 or 3.

2: **uplo – string**

Indicates whether the upper or lower triangular part of A is stored and how B has been factorized.

uplo = 'U'

The upper triangular part of A is stored and $B = U^{\mathrm{T}}U$.

uplo = 'L'

The lower triangular part of A is stored and $B = LL^{\mathrm{T}}$.

Constraint: **uplo** = 'U' or 'L'.

3: **a**(**lda**,$*$) **– double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The n by n symmetric matrix A.

If **uplo** = 'U', the upper triangular part of A must be stored and the elements of the array below the diagonal are not referenced.

If **uplo** = 'L', the lower triangular part of A must be stored and the elements of the array above the diagonal are not referenced.

4: **b**(**ldb**,$*$) **– double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The Cholesky factor of B as specified by **uplo** and returned by f07fd.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The second dimension of the array **a** The second dimension of the array **b**.

n, the order of the matrices A and B.

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldb

5.4 Output Parameters

1: **a**(**lda**,$*$) **– double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The upper or lower triangle of **a** contains the corresponding upper or lower triangle of C as specified by **itype** and **uplo**.

2: **info – int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **itype**, 2: **uplo**, 3: **n**, 4: **a**, 5: **lda**, 6: **b**, 7: **ldb**, 8: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

Forming the reduced matrix C is a stable procedure. However it involves implicit multiplication by B^{-1} (if **itype** $= 1$) or B (if **itype** $= 2$ or 3). When f08se is used as a step in the computation of eigenvalues and eigenvectors of the original problem, there may be a significant loss of accuracy if B is ill-conditioned with respect to inversion. See the document for f08sa for further details.

8 Further Comments

The total number of floating-point operations is approximately n^3.

The complex analogue of this function is f08ss.

9 Example

```
itype = int32(1);
uplo = 'L';
a = [0.24, 0, 0, 0;
     0.39, -0.11, 0, 0;
     0.42, 0.79, -0.25, 0;
     -0.16, 0.63, 0.48, -0.03];
b = [2.039607805437114, 0, 0, 0;
     -1.529705854077835, 1.640121946685673, 0, 0;
     0.2745625891934577, -0.2499814119483738, 0.7887488055748053, 0;
          -0.04902903378454601,  0.6188564222624378,  0.6442661302310234,
0.6160633375780701];
[aOut, info] = f08se(itype, uplo, a, b)
```

```
aOut =
    0.0577         0         0         0
    0.1704    0.2268         0         0
    0.2950    0.8667   -0.0500         0
   -0.6024   -0.6159    0.3972   -1.6875
info =
           0
```